Crossflow Instability in a Spinning Disk Boundary Layer

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Nomenclature

D = diameter of a disk
k = constant
N = spinning speed of a disk, rpm
n = number of crossflow vortices
Re = Reynolds number based on spinning speed
r = distance along radius
Tu = turbulence intensity
U = velocity
u = circumferential velocity in the boundary layer
v = radial velocity in the boundary layer
x = coordinate axis in freestream direction
y = coordinate axis in azimuthal direction
z = coordinate axis normal to the wall
λ = wavelength of disturbances
χ = crossflow parameter

Subscripts

c = critical condition
d = deformation condition
t = transition condition
I = primary disturbance
II = secondary disturbance
β = constant
δ_* = displacement thickness
θ = momentum thickness of boundary layer
ν = dynamic viscosity coefficient
ρ = air density
ω = angular velocity of a spinning disk

Introduction

By controlling the boundary layer and delaying the transition on a swept wing, aircraft drag can be reduced. This can lead to a tremendous amount of total aircraft fuel saving.[1] Because the transition mechanism of a swept wing is similar to that of a spinning disk, boundary layer on a spinning disk has been investigated by many scientists[2-4] in order to study the fundamental mechanism of the three-dimensional boundary-layer transition process. Other practical interests in this fundamental subject centers on the design of a magnetic memory disk for a computer and the transition control of the boundary layer for an advanced propeller, either for aircrafts or ships. In any of these cases, the prediction of transition by numerical method and the understanding of the process of transitional by experimental method are essential to improve the performance of such devices.

It has been made clear by the pioneering investigation of Gregory et al.,[3] both in experimental and theoretical research, that around 30 disturbances over a rotating disk spiraling outward at an angle of about 14 deg. In this research, the difference between experiment and calculation for the critical condition was considerably large. Later, this flowfield was studied by several investigators.[5-7] However, in spite of it, there are still many unsolved problems, and some confusion exists over several features like number of the spiral disturbances, which can be attributed to the different measuring techniques used.

There are several methods measuring the number of the disturbances, like visualization (wall or flow itself), acoustics, or hot-wire techniques. In any case, it is essential to know more detailed features of the disturbance in advance. For instance, one cannot measure the disturbance number using a hot-wire method without knowing the phase velocity of it. Until Ref. 8 shows the structural view of the spiral disturbance, or measures the spatial phase velocity of the disturbance relative to the disk wall, terms like corotating spiral *vortex* and the wall *fixed* nature of the vortex are only the numerical assumptions. To tell the truth, they start to have certain phase velocity at the later stage of the transition.[9]

In Ref. 8 corotating mechanism of the disturbance was clearly visualized by the light sheet method, and the wall fixed nature of the spiral vortices are clarified by two different methods. The existing controversy over the value of the critical Reynolds number in previous investigators both in numerical and experimental research had been one of the important research objectives. Kobayashi et al.[10] introduced effects of streamline curvature and Coriolis force in linear stability calculation and obtained a much more realistic value for the critical Reynolds number. This analysis showed that the inclusion of δ^*/r order terms, namely inclusion of the previously mentioned effects, serve to stabilize the boundary layer considerably. Malik et al.[11] successively calculated the same effect and applied it to the e^Nth method. Discrepancies in experimental critical Reynolds numbers, in our view, can be attributed to the detecting techniques and the performance of the equipment used.

The objective of this Note is to make the transition process clearer by investigating the flowfield using an effective visualization technique that enables us to study three-dimensional real-time transition process in detail.

Experimental Method

The experiment was carried out on two spinning disks that are 400 and 600 mm in diameter, made of 8-mm-thick aluminum, and mounted to a horizontal axis. The spinning speed is continuously varied up to 5000 rpm. Boundary layers appearing on the spinning disks were measured by a hot-wire technique and smoke visualization method. We coated titanium tetrachloride liquid on the black painted disk surface in advance. The flow patterns could be visualized as a motion of white smoke producing from the surface. It was certified that, at the beginning, the white smoke represented the flow near the wall; namely, high-momentum boundary-layer flow in spinning body cases. They were observed stroboscopically and photographed. In order to take a cross-sectional view of the visualized boundary layer, light-sheet illumination with a close-up camera system was used.

Results

Figures 1a–1c show the visualized flow pattern of the boundary-layer flow over a disk in three different conditions. Figure 1a shows full laminar state where one can see no noticeable disturbances, whereas in Figs. 1b and 1c, the conditions are quite different. It can be distinctly observed that two critical radii are indicated separating radially laminar, transitional, and turbulent regions and that crossflow vortices are appearing spirally in the transition region. The number n of the vortices in the figures are 33 and 35. However, if we investigate the transition region in more detail by close-up camera, as seen in Fig. 2a, new instability[10] appears on each spiral vortex near the end of transition as a regularly arranged ringlike shape. Although there appears to be two different types of instabilities, we call the spiral vortices (crossflow instability) the primary instability and the new ringlike disturbances the secondary instability. In Fig. 2a, one can also

Presented as Paper 87-1340 at the AIAA Fluid Dynamics, Plasma Dynamics, and Lasers Conference, Honolulu, HI, June 8–10, 1987; received Feb. 3, 1992; revision received May 4, 1992; accepted for publication May 12, 1992.

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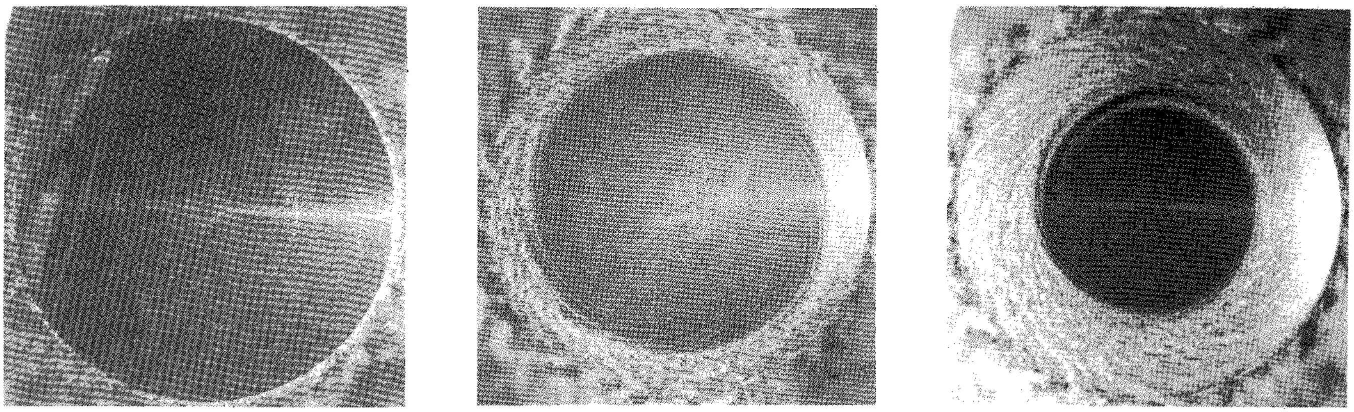
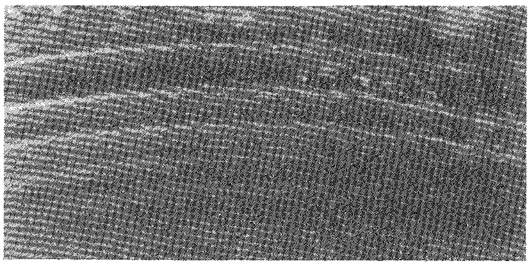
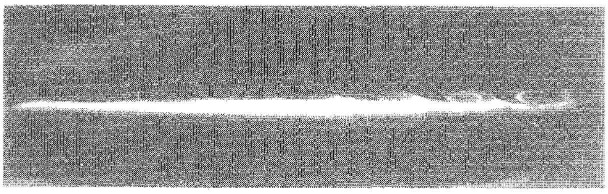


a) N = 900 rpm, D = 400 mm   b) N = 1800 rpm, D = 400 mm   c) N = 5000 rpm, D = 400 mm

Fig. 1 Visualization of a spinning disk boundary layer.



a) Secondary instability



b) Cross-sectional view of the crossflow vortices

Fig. 2 Structure of spiral streaks.

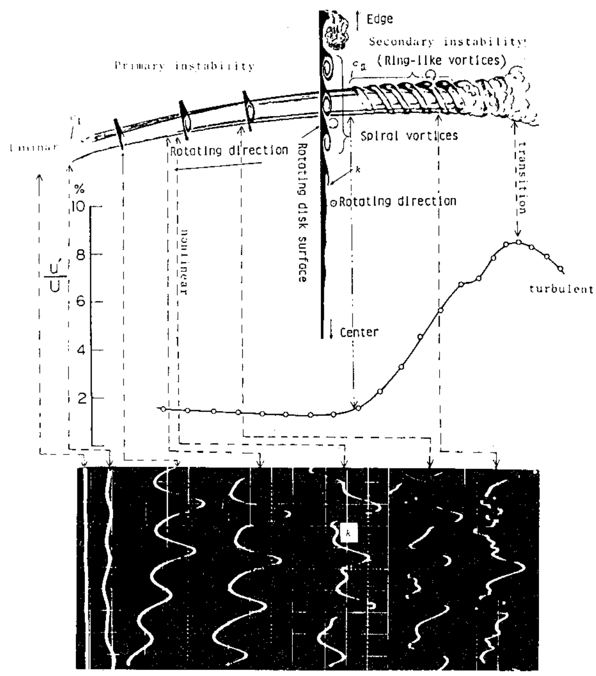
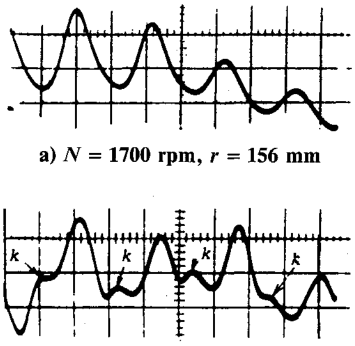


Fig. 3 Turbulence intensity distribution.

notice one to three lines in each spiral vortex. These can be some indication of the structural deformation inside the vortex; however, we cannot explain anything at present. In Fig. 2b, the cross-sectional view of the boundary layer is visualized and corotating crossflow vortices are observed. Comparing this figure with Fig. 2a, one now knows the meaning of these lines in each spiral vortex. These represent the turning over corner of the smoke pattern inside each vortex if observed from above. The relative relation of these two pictures can be seen schematically in Fig. 3. Figure 2a does not give us the detailed structure of the secondary instability. However, experimental results from another flowfield,[12] which is also the crossflow field in the case of a spinning sphere, helps us in understanding this. As a result, we come to know that the secondary instability is also corotating vortices[13] appearing on the surface of each primary disturbance, crossflow vortex. The difference in physical size between primary and secondary disturbances is about 6 to 10. From the point of transition mechanism, it is intersting to know the nature of this secondary instability in more detail. In order to study stationary disturbances (primary instability) in the crossflow field, a spinning disk is advantageous while a hot-wire probe acts as a shooting probe because the flowfield itself moves relative to a fixed hot wire. However, in order to study the secondary instability, the rotating disk is disadvantageous because one cannot distinguish two different motions. Therefore, in order to study the nature of this secondary instability in more detail, it is suggested to look for another crossflow field. Here, we have studied the crossflow field over a swept cylinder[14] and concave-convex curved wall with a sweep angle[15] using a hot-wire anemometer, and we detected that the secondary disturbance has phase velocity and the possible driving instability source is the inflectional instability, which is created by the primary disturbance motion. The motion of the spiral vortex in the y–z plane carries the fluids very near the wall up to the outer edge of the boundary layer where the velocity of the surrounding fluids are greatly different, and as a result, velocity profile involves a point of inflection near the outer edge of the boundary layer.

Figure 3 also shows the turbulence intensity distribution in the streamwise direction with the sketch of transition process of the boundary layer in the case of a general crossflow field. From this, one can notice that the transition process is not simple as had been considered. Because the flowfield has a considerably complicated nature, investigating the flowfield by one-point sensors, such as a hot-wire probe, and measuring the Eulerien properties sometimes give misunderstandings in analyzing the obtained data. In Ref. 16, a wave doubling phenomenon was reported in the case of another crossflow field, and the physical meaning of the phenomenon is not yet known. In Figs. 4a and 4b, measured wave deformation in the case of a rotating disk is shown. One cycle of the wave repre-



a) N = 1700 rpm, r = 156 mm

b) N = 1700 rpm, r = 161 mm

Fig. 4 Deformation of a hot-wire signal.

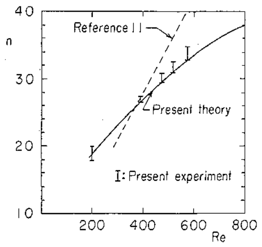


Fig. 5 Number of the crossflow vortices.

sents one crossflow vortex. As the vortex develops, a little kink at the valley of one cycle of the wave will appear, as shown in Fig 4b, and it will become large later on. The physical mechanism of this phenomenon is explained in Ref. 17, and the idea is drawn in Fig. 3. In the figure, different hot-wire traces are placed relative to the transition process. From this, it is obvious that the wave doubling in a hot-wire trace corresponds to the deformation of the primary vortex. Namely, regarding the smoke region in each vortex as high velocity fluids relative to the surrounding ones, the tip of the smoke pattern, indicated as k, corresponds to the little kink k in a hot-wire trace. Therefore, the wave doubling means the lifting up of the bottom (of the boundary layer) flow by rotation motion of the vortex.

The important information in Fig. 3 is that the rapid increase in turbulence intensity corresponds not to the critical point of primary instability, but to that of the secondary one. It is also interesting to know that the turbulence level at the fully turbulent stage has a lower level than the final stage of the transition and it has a tendency to decrease. This can be explained by the fact that the spatial turbulence intensity per unit volume decreases as the boundary layer becomes fully turbulent owing to the broadening of boundary layer thickness. Another important fact in Fig. 3 is that a hot-wire signal at the secondary instability stage looks irregular, and at a first glance, one might regard this as a turbulent stage. However, this stage is still a transition stage where two regular disturbances are appearing and interacting. Observation of single data, such as hot-wire signal, sometimes leads to misreading the phenomena. Coupled use or multiple use of different measuring techniques is essential.

In Ref. 11, the number n of primary instability is given by

$$n = \beta Re \tag{1}$$

where β is certain constant and Re is the spin Reynolds number. This shows the linear dependence of n to Re. However, as shown in Fig. 5, it is not likely to be so; n increases at a low Reynolds number, but seens to become saturated as the Reynolds number becomes large. Since the size of the primary vortices roughly depends on the boundary-layer thickness, we should suggest that

$$n = k\sqrt{Re} \tag{2}$$

where k is a constant in the order of $1.3 \sim 1.4$.

In order to compare the onset condition of crossflow instability with other crossflow fields, twisted three-dimensional boundary-layer profiles were measured using a single hot-wire probe with rotary mechanism in its axis normal to disk surface. Using obtained crossflow velocity profiles, crossflow parameter χ suggested by Poll[18] was calculated. Critical value for χ was about 250, which is a slightly higher value than obtained for a yewed cylinder.[18]

Conclusions

The transition process of the three-dimensional boundary layer on a spinning disk is studied experimentally in detail. The experiments clearly showed the existence of two different instabilities in the transition region, and the physical structure of the wave doubling phenomena was explained. It is also clarified that the secondary instability has the major role to the turbulent transition process than the primary instability. Wide range experiment in spin Reynolds number Re obtained new relation that the number of the primary vortices increases with square root of Re.

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